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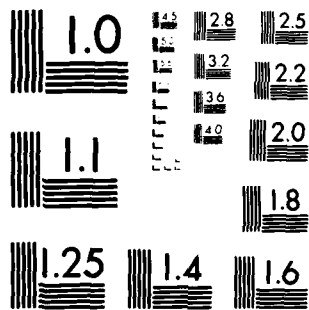
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INPUT-OUTPUT STABILITY ANALYSIS WITH MAGNETIC
HYSTERESIS NON-LINEARITY - A CLASS OF MULTIPLIERS

M. G. Safonov* and K. Karimlou*

Department of Electrical Engineering-Systems
University of Southern California
Los Angeles, CA 90089-0781Abstract

A class of positive real multipliers is obtained to establish frequency domain conditions for stability of feedback systems containing ferromagnetic hysteresis non-linearity.

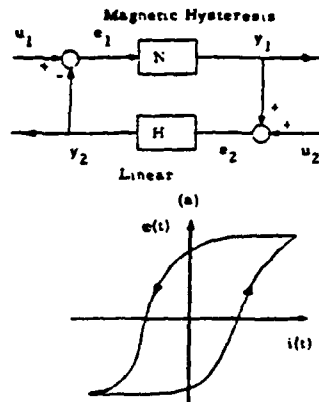
I. Introduction

Popov Criterion [1] and its extensions, such as derivation of more general classes of multipliers [2], [3], consider non-linear elements that are memoryless and pass through the origin, i.e., $\phi(0) = 0$. An important class of non-linearities, ferromagnetic hysteresis, is neither non-dynamic nor passes through the origin. Therefore, to analyze the stability of systems containing this type of non-linearity, appropriate modifications to Popov's approach should be made.

Published material to address this problem is scarce. Authors in [4] obtained multipliers of the form $1+qj\omega$, $q \geq 0$ for finite gain L_2 -stability

of magnetic hysteresis feedback systems. This paper generalizes the results of [4]. The only other work known to us that addresses the problem of stability of magnetic hysteresis systems is by Lecoq and Hopkin [5], where by letting the derivative of their input signals to belong to exponentially weighted L_2 -spaces, they obtained similar multipliers, $1+qj\omega$, for bounded input-bounded output stability of hysteresis systems. For stability of systems with other types of hysteresis non-linearity, such as backlash and relay, see Yakubovich [6], Hsu and Meyer [7], Kodama and Shirakawa [8], Maeda, Ikeda, and Kodama [9], [10].

In the present paper we obtain a general class of positive real multipliers for the stability analysis of feedback systems of the type shown in Fig. (1a), where N is a ferromagnetic hysteresis non-linearity and H is a linear element. The analysis is done by substitution of the model for the hysteresis by Chua and Strossmoe [11]. Then the concepts of positivity and passivity are utilized to derive a frequency



(b) Ferromagnetic Hysteresis Loop.

Fig. 1

domain conditions on the linear element H for finite gain L_2 -stability of the feedback system.

II. Hysteresis Modeling

The model for ferromagnetic hysteresis of Chua and Strossmoe [11] is given by

$$\frac{dy}{dt} = g\phi[x(t)] - f\phi[y(t)] \quad (2.1)^1$$

where $x(t)$ and $y(t)$ are real-valued, continuous input and output signals of the hysteresis non-linearity representing the current $i(t)$ and the flux linkage $\phi(t)$ of an inductor (transformer); and g and f are strictly monotonically increasing, differentiable, onto functions enjoying the important property of

$$g(0) = f(0) = 0$$

Equation (2.1) models the behavior of ferromagnetic hysteresis successfully and with very good accuracy. It predicts the expansion of the area of the hysteresis loop with increasing frequency and predicts minor hysteresis loops such as commonly occur when a d-c plus periodic input is applied.

¹o: Functional Composition.

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After plotting the hysteresis loop for a convenient signal, simple procedures are given in [11] to determine g and f for that loop. When non-linear functions g and f are determined, they can be substituted in the model of equation (2.1) to predict, with good accuracy, the hysteresis shape and/or its output for any arbitrary input. For further detail and examples see [11].

III. Mathematical Preliminaries

Definition (3.1) [12]: Let $H: L_{2e} \rightarrow L_{2e}$.

Then H is passive iff there exists some constant $\theta \in \mathbb{R}$ such that

$$\langle Hx, x \rangle_T \geq \theta \quad \forall x \in L_{2e} \\ \forall T \in J$$

Definition (3.2) [12]: Let $H: L_{2e} \rightarrow L_{2e}$.

Then H is strictly passive iff there exists $\delta > 0$ and some constant $\theta \in \mathbb{R}$ such that

$$\langle Hx, x \rangle_T \geq \delta \|x_T\|^2 + \theta \quad \forall x \in L_{2e} \\ \forall T \in J$$

Definition (3.3) [13]: The feedback system of Fig. (1a) is said to be finite gain L_2 -stable if

$$a) e_1, e_2, y_1, y_2 \in L_2 \quad \forall u_1, u_2 \in L_2$$

$$b) \text{ There exists constants } \rho_1 \text{ and } \rho_2 \text{ such that}$$

$$\|e_1\|, \|e_2\|, \|y_1\|, \|y_2\| \leq \rho_1 \|u_1\| + \rho_2 \|u_2\| \\ \forall u_1, u_2 \in L_2$$

In the following well-known theorem, the concept of passivity is used to establish finite gain L_2 -stability of the feedback system shown in Fig. (1a), where N and H are considered to be operators in the general sense.

Theorem (3.1): Consider the feedback system shown in Fig. (1a)

$$e_1 = u_1 - He_2$$

$$e_2 = u_2 + Ne_1$$

where $H, N: L_{2e} \rightarrow L_{2e}$. Assume that for any $u_1, u_2 \in L_2$ there are solutions $e_1, e_2 \in L_{2e}$. Suppose that there are real constants ν, δ , and ϵ such that

$$\|Hx\|_T \leq \nu \|x\|_T \quad (3.1)$$

$$\langle Hx, x \rangle_T \geq \delta \|x\|_T^2 \quad (3.2)$$

$$\langle x, Nx \rangle_T \geq \epsilon \|Nx\|_T^2 \quad (3.3) \\ \forall x \in L_{2e}, \forall T \in J$$

Under these conditions if

$$\delta + \epsilon > 0 \quad (3.4)$$

then the feedback system is finite gain

L_2 -stable.

Proof: See for example [12].

Definition (3.4) [12]: Let $H: L_2 \rightarrow L_2$. Then H is positive iff

$$\langle Hx, x \rangle \geq 0, \quad \forall x \in L_2$$

Note that unlike passivity, which is defined on L_{2e} , positivity is defined on L_2 .

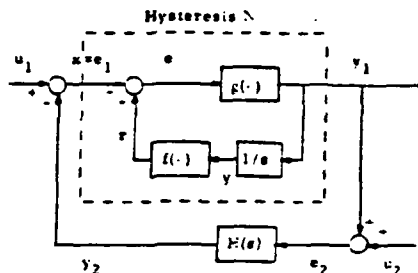


Fig. 2

Table 1
Notations

Symbol	Meaning
\mathbb{R}, \mathbb{R}_+	Field of real and positive real numbers.
L_1	The space of signals such that $\int_{-\infty}^{\infty} x(t) dt$ exists.
L_∞	The space of bounded signals.
L_{2e}	The space of signals which are square integrable on every bounded interval $[0, T]$ [14].
L_2	The Hilbert space of signals which are square integrable on $(-\infty, \infty)$ with inner product $\langle x, y \rangle$.
$\langle x, y \rangle$	$\int_{-\infty}^{\infty} y^*(t) x(t) dt$
$\ x\ $	$\sqrt{\langle x, x \rangle}$
$\langle x, y \rangle_T$	$\int_0^T y^*(t) x(t) dt$
$\ x\ _T$	$\sqrt{\langle x, x \rangle_T}$
x_T	Truncated x [15].
J	Set of instances of time of interest.
\mathcal{A}	Convolution Algebra [12].
$\text{Re}(\cdot)$	Real part of a complex quantity.

Next, a positivity result for a cascade combination of a linear and a non-linear operator is presented.

Theorem (3.2) [13]: Assume $G: L_2 \rightarrow L_2$ is a linear operator, with $G(j\omega) \in L_\infty$, which maps $x \in L_2$ into the element in L_2 , and $\Phi: L_2 \rightarrow L_2$ is a monotone non-decreasing non-linear operator. Let $G(j\omega)$ be given by the Fourier-Stieltjes integral

$$G(j\omega) = 1 - \int_{-\infty}^{\infty} e^{-j\omega\tau} dV(\tau) \quad (3.4)$$

where $V(\tau)$ is any monotone non-decreasing function of total variation less than or equal to unity. Then $G\Phi$ is a positive operator on L_2 .

IV. Main Results

Substitution of the model given by equation (2.1) for the hysteresis non-linearity N gives the feedback system of Fig. (2). Note that although the standard magnetic hysteresis non-linearity, as shown in Fig. (1b), is the plot of flux linkage ($\Phi(t)$) vs. current ($i(t)$) of an inductor (transformer), but from circuit analysis point of view, the input and output of the model replaced for N as shown in Fig. (2) are current through and voltage across the inductor (transformer).²

The class of positive real multipliers is defined next.

Definition (4.1): \mathcal{M} is the class of multipliers $M(s)$ of the form:

$$M(s) = \beta_0(s+z_0) + \sum_{i=1}^n \frac{\alpha_i(s+z_0)}{s+p_i}$$

where: $\beta_0 = 0$ or 1 ; $z_0, \alpha_i \geq 0$; $\alpha_i > 0$, $\forall i$, $i \neq 1$; $p_i > 0$, $\forall i$; $z_0 < \min \{p_i\}$.

\mathcal{M} is similar to the class of multipliers studied in [2] and [3] in stability analysis of standard Popov type feedback systems with monotone non-decreasing non-linearity. It is shown that [2], $M(s)$ can also be represented in the form

$$M(s) = \frac{\prod_{i=1}^k (s+\lambda_i)}{\prod_{i=1}^l (s+\eta_i)}$$

where: $l=k$ or $k+1$; $0 < \lambda_1 < \eta_1 < \lambda_2 < \eta_2 < \dots$.

The above representation implies that $M(s)$ consists of a combination of real valued poles and zeroes in the left half plane, where the first singularity is a zero, and the poles and zeroes alternate.

Next, the main stability result is presented.

Theorem (4.1): Consider the feedback system of Fig. (2), where $h(t) \in L_1(R_+)$ ³, and $\dot{h}(t) \in \mathcal{A}$. Assume that for any $u_1, u_2 \in L_2$, there are solutions $e_1, e_2, y_1, y_2 \in L_2$. If for some constant $\delta > 0$ and $M \in \mathcal{M}$

$$\inf \operatorname{Re} \{M(j\omega)H(j\omega)\} = \delta > 0, \quad \forall \omega \geq 0 \quad (4.1)$$

then $\forall u_1, \dot{u}_1, u_2 \in L_2$.

²The voltage across an inductor (transformer) is proportional to the rate of change of the flux linkage, the constant of proportionality being the number of turns.

³ $h(t)$ is impulse response of $H(s)$.

- a) (i) $e_1, \dot{e}_1, e_2, y_1, y_2, \dot{y}_2 \in L_2$;
(ii) There exists constants ρ_1, ρ'_1 , and ρ_2 such that

$$\|e_1\|, \|\dot{e}_1\|, \|e_2\|, \|y_1\|, \|y_2\|, \|\dot{y}_2\| \leq \rho_1 \|u_1\| + \rho'_1 \|\dot{u}_1\| + \rho_2 \|u_2\|$$

i. e., finite gain L_2 -stability.

- b) $e_1, e_2, y_1, y_2 \in L_\infty$ are continuous, and go to zero as $t \rightarrow \infty$.

Proof: By taking the truncated inner-products of each branch of the feedback system and utilizing Theorem (3.2), positivity and consequently passivity of each branch is proven. Then using Theorem (3.1), stability of the system is deduced.

For detail of the proof, see Appendix.

Upper and lower bounds of non-linearities g and f can be taken into consideration to obtain less conservative classes of linear element $H(s)$. As an example, an upper bound of g is exploited in the following Corollary:

Corollary (4.1): Consider the feedback system of Fig. (2), where $g \in \text{sector}(0, k)$, $\frac{dg(x)}{dx} \leq k$, $h(t) \in L_1(R_+)$, $\dot{h}(t) \in \mathcal{A}$. Assume that for any $u_1, u_2 \in L_2$, there exists solutions $e_1, e_2, y_1, y_2 \in L_2$. If for some constant $\delta > 0$ and $M \in \mathcal{M}$

$$\operatorname{Re} \left\{ M(j\omega) \left[H(j\omega) + \frac{1}{k} \right] \right\} = \delta > 0, \quad \forall \omega \geq 0 \quad (4.2)$$

then $\forall u_1, \dot{u}_1, u_2, \dot{u}_2 \in L_2$ conclusions of Theorem (4.1) hold.

Proof: See Appendix.

V. Conclusion

A class of positive real multipliers is obtained to establish frequency domain conditions for stability of feedback systems containing magnetic hysteresis non-linearity. To obtain the results, the model of Chua and Stromsmoe [11] for hysteresis is employed and the concepts of positivity and passivity are utilized.

VI. Appendix

To simplify the proof of Theorem (4.1) and Corollary (4.1), the following two lemmas will be proved first.

Lemma (A.1): Let $g \in \text{sector}(0, \infty)$, $\frac{dg(x)}{dx} > 0$, and $M \in \mathcal{M}$. Then the system of Fig. (A.1) is passive.

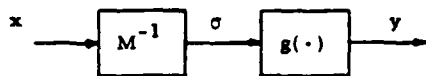


Fig. (A.1)

Proof: For all $x \in L_{2e}$ and $T \geq 0$
 $\langle x, y \rangle_T = \langle x, gM^{-1}x \rangle_T$. Let

$$\begin{aligned} M^{-1}x &= \hat{x}, \\ &= \langle M\hat{x}, g\hat{x} \rangle_T \\ &= \langle \theta_0 S \hat{x}, g\hat{x} \rangle_T + \langle \theta_0 z_0 \hat{x}, g\hat{x} \rangle_T \\ &\quad + \langle \Sigma(S)\hat{x}, g\hat{x} \rangle_T \quad (A.1) \end{aligned}$$

where $S\hat{x} = \frac{d}{dx}(\hat{x})$, $g\hat{x} = g(\hat{x})$, and $\Sigma(S)$ is the rational part of $M(s)$. The first two inner-products of equation (A.1) are passive [4].

$$\begin{aligned} \langle \Sigma(S)\hat{x}, g\hat{x} \rangle_T &= \langle P_T \Sigma(S) P_T \hat{x}, P_T g P_T \hat{x} \rangle \\ &= \langle P_T \hat{x}, \Sigma^*(S) P_T^* g P_T \hat{x} \rangle \\ &= \langle P_T \hat{x}, \Sigma^*(S) g P_T \hat{x} \rangle \end{aligned}$$

But $P_T \hat{x} \in L_2$, thus, $\Sigma^*(S)$ is a mapping of $L_2 \rightarrow L_2$. Therefore, by Theorem (3.2)

$$\langle P_T \hat{x}, \Sigma^*(S) g P_T \hat{x} \rangle \geq 0$$

which implies passivity of Fig. (A.1). \square

Lemma (A.2): Let $f(\cdot) \in (0, \infty)$, $\frac{df(x)}{dx} > 0$, and $M \in \mathcal{M}$. Then the system of Fig. (A.2) is passive. \square



Fig. (A.2)

Proof: For all $x(t) \in L_2$ and $T \geq 0$,
 $\langle x, y \rangle_T = \langle x, Mf \frac{1}{s} x \rangle_T$ where $\frac{1}{s} x = \int x dt$.
 Let $\frac{1}{s} x = \hat{x}$, then $\hat{x} \in L_{2e}$ since $x \in L_{2e}$.

$$\begin{aligned} &= \langle S\hat{x}, M\hat{x} \rangle_T \\ &= \langle S\hat{x}, \theta_0 S\hat{x} \rangle_T + \langle S\hat{x}, \theta_0 z_0 \hat{x} \rangle_T \\ &\quad + \langle S\hat{x}, \Sigma(S)\hat{x} \rangle_T \quad (A.2) \end{aligned}$$

The first two inner-products of equation (A.2) are passive [4].

$$\begin{aligned} \langle S\hat{x}, \Sigma(S)\hat{x} \rangle_T &= \langle x, \Sigma(S) f \frac{1}{s} x \rangle_T \\ &= \langle P_T x, P_T \Sigma(S) f \frac{1}{s} P_T x \rangle \end{aligned}$$

$$\begin{aligned} \text{Let } \bar{x}_T &= \frac{1}{s} P_T x, \text{ then } S\bar{x}_T = P_T x \in L_2 \\ &= \langle S\bar{x}_T, P_T \Sigma(S) f \bar{x}_T \rangle \\ &= \langle P_T S \bar{x}_T, \Sigma(S) f \bar{x}_T \rangle \end{aligned}$$

$$\begin{aligned} \text{But } P_T S \bar{x}_T &= P_T x = S\bar{x}_T \\ &= \langle S\bar{x}_T, \Sigma(S) f \bar{x}_T \rangle \\ &= \langle \Sigma^*(S) S \bar{x}_T, f \bar{x}_T \rangle \end{aligned}$$

$$\begin{aligned} \text{Substitution of } \Sigma^*(S) \text{ implies that: } S\bar{x}_T(S) &= \sum_{i=1}^n \alpha_i S \\ + \hat{\Sigma}^*(S) \text{ where } \hat{\Sigma}^*(S) &\triangleq \sum_{i=1}^n \frac{\alpha_i (p_i - z_0)(-S)}{-S + p_i}. \text{ Then} \\ &= \langle \sum_{i=1}^n \alpha_i S \bar{x}_T, f \bar{x}_T \rangle + \langle \hat{\Sigma}^*(S) \bar{x}_T, f \bar{x}_T \rangle \end{aligned}$$

The first inner-product is passive [4], while the second one is positive, by Theorem (3.2). Thus the passivity of Fig. (A.2) follows. \square

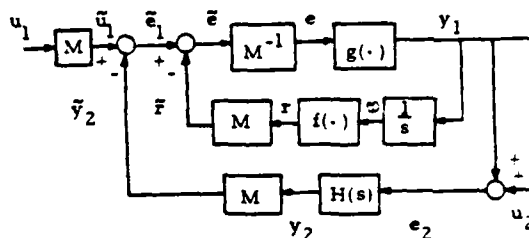


Fig. (A.3)

Proof of Theorem (4.1): By inclusion of the multiplier $M(s)$, transform the feedback system of Fig. (2) to the one shown in Fig. (A.3). For the feed forward block:

$$\langle y_1, \bar{e}_1 \rangle_T = \langle y_1, \bar{e} \rangle_T + \langle y_1, \bar{r} \rangle_T$$

By Lemmas (A.1) and (A.2), the inner products on the right-hand side of the above equality are passive. Thus, the feed forward block of Fig. (A.3) is passive.

The feedback block of the above system is strictly passive with finite gain if the conditions set on $H(s)$ and inequality (4.1) are satisfied.

Since $\bar{u}_1 = m(t) * u_1$, where $m(t)$ is Inverse Laplace Transform of $M(s)$, and $u_1, \dot{u}_1 \in L_2$, thus $\bar{u}_1(t) \in L_2$. Therefore, by Theorem (3.1), the feedback system of Fig. (A.3) is finite gain L_2 -stable, i.e., $\bar{e}_1, e_2, y_1, \bar{y}_2 \in L_2$ and $\|\bar{e}_1\|, \|e_2\|, \|y_1\|, \|\bar{y}_2\| \leq \rho_1 \|\bar{u}_1(t)\| + \rho_2 \|u_2\|$

But

$$\begin{aligned} \|\bar{u}_1(t)\| &= \|m(t) * u_1(t)\| \\ &\leq \rho_1 \|u_1\| + \rho_1' \|\dot{u}_1\| \end{aligned}$$

The last inequality is obtained by substitution of $m(t)$ and simplification of the norm. Thus:

$$\|\bar{e}_1\|, \|e_2\|, \|y_1\|, \|\bar{y}_2\| \leq \rho_1 \|u_1\| + \rho_1' \|\dot{u}_1\| + \rho_2 \|u_2\|$$

From Fig. (A.3), $y_2 = m^{-1}(t) * \tilde{y}_2 \cdot m^{-1}(t) \in L_1$ and $\tilde{y}_2 \in L_2$ implies that $y_2(t), \dot{y}_2(t) \in L_2$ [12, Appendix C]. Furthermore, $\|y_2\|_{L_2} = \|m^{-1}(t) * \tilde{y}_2(t)\|_{L_2} \leq \|m^{-1}(t)\|_{L_1} \|\tilde{y}_2(t)\|_{L_2}$ [12, App. C]. $m^{-1}(t) \in L_1$ implies that $\|m^{-1}(t)\|_{L_1} = C$, a constant. Thus

$$\begin{aligned} \|y_2(t)\|_{L_2} &\leq C \|\tilde{y}_2\|_{L_2} \\ &= C \rho_1 \|u_1\| + C \rho_1' \|\dot{u}_1\| + C \rho_2 \|u_2\| \end{aligned}$$

On the other hand, $\dot{y}_2 = \dot{m}^{-1}(t) * \tilde{y}_2$. Similar upper bounds for $\|\dot{y}_2\|$ are immediate, since $m^{-1}(t) \in L_1$.

Similarly, $e_1 = m^{-1}(t) * \tilde{e}_1$. Thus, similar conclusions as y_2 for e_1 follow immediately.

b) $y_2, \dot{y}_2 \in L_2$ and $e_1, \dot{e}_1 \in L_2$ imply that $y, e \in L_\infty$ are continuous, and go to zero as $t \rightarrow \infty$ [13]. Since the model, i. e., equation (2.1), is a continuous mapping from input to output [11], therefore, $e_1 \in L_\infty$ and $e_1(t) \rightarrow 0$ as $t \rightarrow \infty$ imply that the same properties hold for $y_1(t)$, i. e., $y_1(t) \in L_\infty$ is continuous, and go to zero as $t \rightarrow \infty$.

Similar conclusions for e_2 are immediate. ■

Proof Outline of Corollary (4.1): Apply a positive feedback of gain $\frac{1}{k}$ around g . To compensate for it, apply a positive feed-forward with gain $\frac{1}{k}$ to $H(s)$. Let $\hat{g} = (g^{-1} - \frac{1}{k})^{-1}$. Then $\hat{g} \in \text{sector } (0, \infty)$, is monotone increasing, and $\hat{g}(0) = 0$. Follow the same procedure as Theorem (4.1). Conclusions are immediate. ■

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